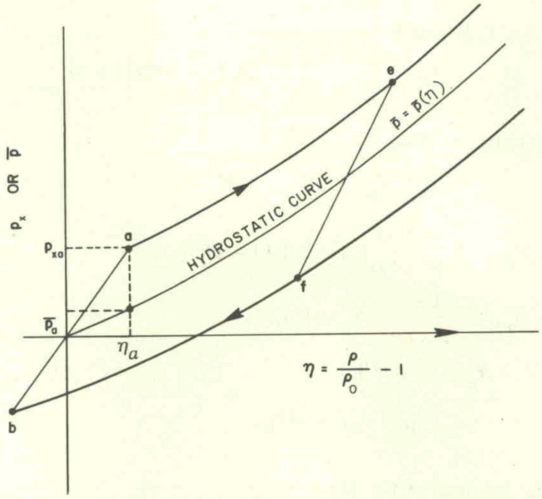

Fig. 1. Elastoplastic Equation of State

where K is bulk modulus, μ is rigidly modulus, ν is Poisson's ratio, $d\epsilon_x = d\rho/\rho$ is incremental strain in the x-direction and ρ is density. For ν = const., which is assumed throughout the following, Eq. (5) can be integrated directly:

$$p_x = 3\bar{p}(1-\nu)/(1+\nu). \tag{6}$$

Since strain is uniaxial,

$$dp_y = (K - 2\mu/3)\, d\epsilon_x \,, \tag{7}$$

and the incremental change in τ is

$$d\tau = (dp_x - dp_y)/2 = 3d\bar{p}(1-2\nu)/2(1+\nu) \tag{8}$$

or

$$\tau = 3\bar{p}(1-2\nu)/2(1+\nu). \tag{9}$$

At *a:*

This is the initial yield point, so

$$Y_a = 2\tau_a = 3\bar{p}_a(1-2\nu)/(1+\nu). \tag{10}$$

Along *ae:*

Equation (4) is integrated to obtain

$$p_x - p_{xa} = \bar{p} - \bar{p}_a + \frac{2}{3}(Y - Y_a). \tag{11}$$

At *e:*

As unloading begins, yield ceases, and

$$Y = Y_e. \tag{12}$$

Along *ef:*

This is the unloading phase; τ diminishes and changes sign, and the material once again behaves elastically. Equation (5) integrates to

$$p_{xe} - p_x = 3(1-\nu)(\bar{p}_e - \bar{p})/(1+\nu).$$

The integral of Eq. (8) is

$$Y_e - 2\tau = 3(\bar{p}_e - \bar{p})(1-2\nu)/(1+\nu).$$

Combining this with Eq. (12) yields

$$Y_e - 2\tau = (p_{xe} - p_x)(1 - 2\nu)/(1 - \nu) \tag{13}$$

At *f:*

This is the unloading yield point;

$$p_y - p_x = Y_f = -2\tau_f \tag{14}$$

and from Eq. (13),

$$Y_e + Y_f = (p_{xe} - p_{xf})(1 - 2\nu)/(1 - \nu)$$

$$p_{xe} - p_{xf} = \frac{1 - \nu}{1 - 2\nu}(Y_e + Y_f). \tag{15}$$

In the special case $Y_f = Y_e$, $\nu = 1/3$,

$$p_{xe} - p_{xf} = 4Y_e.$$

Along *fb:*

Here we have the integral of Eq. (4):

$$p_x - p_{xf} = \bar{p} - \bar{p}_f - \frac{2}{3}(Y - Y_f). \tag{16}$$

At *b:*

The resolved shear stress, calculated elastically, is equal to half the yield stress:

$$Y_b = 2\tau_b = 3\bar{p}_b(1 - 2\nu)/(1 + \nu)\,. \tag{17}$$

Equations (5) through (17) provide means for calculating the stress-strain cycle of Fig. 1 if $\bar{p}(\rho)$, $Y(\bar{p})$, and ν are known. The extent to which ν may vary during such a cycle is presently unknown.

The value of the yield strength is assumed to vary with the pressure according to the relation

$$Y = Y_0 + M(\bar{p} - \bar{p}_a)\,. \tag{18}$$

The mean pressure is assumed to be related to the density by the expression

$$\bar{p} = A\eta + B\eta^2 + C\eta^3 \tag{19}$$

where $\eta = (\rho/\rho_0) - 1$ and ρ_0 is density at $\bar{p} = 0$. The sound speed is

$$c = \sqrt{dp_x/d\rho} = [-3V^2(1 - \nu)(d\bar{p}/dV)/(1 + \nu)]^{1/2} \tag{20}$$

Sound speed is assumed constant, $c = c_0$, along *ba* of Fig. 1. Integrating Eq. (20) under this assumption yields

$$\bar{p} = [c_0^2(1 + \nu)/3(1 - \nu)](V^{-1} - V_0^{-1}). \tag{21}$$

Then, at *a*,

$$\eta_a = 3\bar{p}_a V_0(1 - \nu)/c_0^2(1 + \nu). \tag{22}$$

Sound speed on the segments *ae*, *fb* of Fig. 1 is defined by the slopes of the curves shown:

$$c_p^2 = dp_x/d\rho = d\bar{p}/d\rho \pm \frac{2}{3}\,dY/d\rho. \tag{23}$$

The values of the coefficients in Eq. (19) are obtained by assuming that Hugoniot equation of state data lie on the upper curve of Fig. 1. Values of Y_0 and M must then be assumed, see Eq. (18). The value of Y_0 corresponds to that obtained statically, and the value of M is esti-